Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

1 - 3 Geometric interpretation
Solve graphically and explain geometrically.

I'm taking a shortcut here. I would like to use **LinearSolve** or similar functions. However, the problems as written are formulated perfectly for cutting and pasting into **Solve**, so that's what I use.

1. $x_1 - 4\,x_2 == 20.1\;;\; 3\,x_1 + 5\,x_2 == 5.9$

```
Solve[x1 - 4 x2 == 20.1 && 3 x1 + 5 x2 == 5.9, {x1, x2}]
```

```
{{x1 → 7.3, x2 → -3.2}}
```

3. $7.2\,x_1 - 3.5\,x_2 == 16.0\;;\; -14.4\,x_1 + 7.0\,x_2 == 31.0$

```
Solve[7.2 x1 - 3.5 x2 == 16.0 && -14.4 x1 + 7.0 x2 == 31.0, {x1, x2}]
```

```
{}
```

Mathematica is telling me there is no solution to the above problem, and the text answer agrees.

4 - 16 Gauss elimination
Solve the following linear systems by Gauss elimination, with partial pivoting if necessary (but without scaling). Show the intermediate steps. Check the result by substitution. If no solution or more than one solution exists, give a reason.

5. $2\,x_1 - 8\,x_2 == -4\;;\; 3\,x_1 + x_2 == 7$

```
Solve[2 x1 - 8 x2 == -4 && 3 x1 + x2 == 7, {x1, x2}]
```

```
{{x1 → 2, x2 → 1}}
```

7. $-3\,x_1 + 6\,x_2 - 9\,x_3 == -46.725\;;$
$x_1 - 4\,x_2 + 3\,x_3 == 19.571\;;\; 2\,x_1 + 5\,x_2 - 7\,x_3 == -20.073$

```
Solve[-3 x1 + 6 x2 - 9 x3 == -46.725 &&
  x1 - 4 x2 + 3 x3 == 19.571 && 2 x1 + 5 x2 - 7 x3 == -20.073, {x1, x2, x3}]
```

```
{{x1 → 3.908, x2 → -1.998, x3 → 2.557}}
```

9. $0 + 6x_2 + 13x_3 = 137.86$;
$6x_1 + 0 - 8x_3 = -85.88$; $13x_1 - 8x_2 + 0 = 178.54$

```
Solve[0 + 6 x2 + 13 x3 == 137.86 &&
  6 x1 + 0 - 8 x3 == -85.88 && 13 x1 - 8 x2 + 0 == 178.54, {x1, x2, x3}]
```

```
{{x1 → 6.78, x2 → -11.3, x3 → 15.82}}
```

11. $3.4x_1 - 6.12x_2 - 2.72x_3 = 0$;
$-x_1 + 1.80x_2 + 0.80x_3 = 0$; $2.7x_1 - 4.86x_2 + 2.16x_3 = 0$

```
Solve[3.4 x1 - 6.12 x2 - 2.72 x3 == 0 &&
  -x1 + 1.80 x2 + 0.80 x3 == 0 && 2.7 x1 - 4.86 x2 + 2.16 x3 == 0, {x1, x2, x3}]
```

Solve:svars: Equations may not give solutions for all "solve" variables. >>

$$\left\{\left\{x_2 \to 0. + 0.555556\, x_1,\ x_3 \to 0. + 6.47025 \times 10^{-17}\, x_1\right\}\right\}$$

```
x1 = 1
```

```
1 (* the value of x1 is arbitrary *)
```

```
x2 = 0.5555555555555556`
```

```
0.555556
```

```
x3 = 0
```

```
0
```

For this one I am following the problem section's instruction to check by substitution.

```
3.4 x1 - 6.12 x2 - 2.72 x3
```

$$-4.44089 \times 10^{-16}$$

```
-x1 + 1.80 x2 + 0.80 x3
```

```
0.
```

```
2.7 x1 - 4.86 x2 + 2.16 x3
```

```
0.
```

$$\frac{3.4}{6.12}$$

0.555556

The content of green and cyan cells argues for a correct solution.

```
13.  0 + 3 x2 + 5 x3 == 1.20736 ;
3 x1 - 4 x2 + 0 == -2.34066 ;  5 x1 + 0 + 6 x3 == -0.329193
```

This one caused some difficulty when the variables from problem 11 would not clear at first. Finally they did. I suspect this is the subscript bogey raising its head. If it is, it's the first difficulty I can attribute to that habit.

```
Clear["Global`*"]
Quit[]
```

```
Solve[0 + 3 x2 + 5 x3 == 1.20736 &&
  3 x1 - 4 x2 + 0 == -2.34066 && 5 x1 + 0 + 6 x3 == -0.329193, {x1, x2, x3}]
```

```
{{x1 → 0.142856, x2 → 0.692307, x3 → -0.173912}}
```

```
15.  0 + 2.2 x2 + 1.5 x3 - 3.3 x4 == -9.30 ;
0.2 x1 + 1.8 x2 + 0 + 4.2 x4 == 9.24 ;
-x1 + 3.1 x2 + 2.5 x3 + 0 == -8.70 ;  0.5 x1 + 0 - 3.8 x3 + 1.5 x4 == 11.94
```

Here I was using the template for x_1, x_2, x_3 and getting only a null answer out of **Solve**. I wondered if the function was choking, until I noticed that my command phrase was not asking for a solution of x_4. **Solve** would only solve the problem if all the variables were asked for.

```
Clear["Global`*"]
```

```
Solve[0 + 2.2 x2 + 1.5 x3 - 3.3 x4 == -9.30 &&
  0.2 x1 + 1.8 x2 + 0 + 4.2 x4 == 9.24 && -x1 + 3.1 x2 + 2.5 x3 + 0 == -8.70 &&
  0.5 x1 + 0 - 3.8 x3 + 1.5 x4 == 11.94, {x1, x2, x3, x4}]
```

$$\left\{\left\{x_1 \to 4.2,\ x_2 \to -2.65512 \times 10^{-16},\ x_3 \to -1.8,\ x_4 \to 2.\right\}\right\}$$

So I finish the problem section without doing anything to familiarize myself with Gauss elimination and pivoting.